Chapter 6

Discounted Cash Flow Valuation

Key Concepts and Skills

Be able to compute:

- the future value of multiple cash flows
- the present value of multiple cash flows
- the future and present value of annuities

Be able to understand:

- the difference between the APR and the EAR
- the difference between "ordinary annuity" and "annuity due"

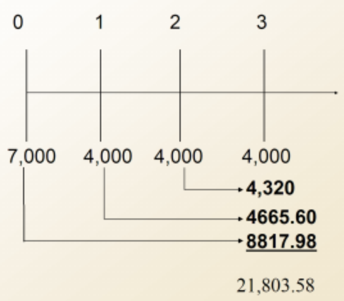
Multiple Cash Flows –Future Value Example 6.1

- You think that you will be able to deposits £4000 at the end of each of the next three years in a bank account paying 8 percent interest. You currently have £7000 in the account? How much will you have in three years? In four years?

Answer 6.1

- Find the value at year 3 of each cash flow and add them together.
  - Today (year 0): FV = $7000(1.08)^3$ = 8,817.98
  - Year 1: FV = $4,000(1.08)^2$ = 4,665.60
  - Year 2: FV = 4,000(1.08) = 4,320
  - Year 3: value = 4,000
  - Total value in 3 years = 8817.98 + 4665.60 + 4320 + 4000 = 21,803.58

Timeline 6.1



Multiple Cash Flows – FV Example 6.2

- Suppose you invest $500 in a mutual (shared, joint) fund today and $600 in one year. If the fund pays 9% annually, how much will you have in two years?

Answer 6.2

- How much will you have in 5 years if you make no further deposits?
- First way:
  - $FV = 500(1.09)^5 + 600(1.09)^4 = 1616.26$
- Second way – use value at year 2:
  - $FV = 1248.05(1.09)^3 = 1616.26$

Multiple Cash Flows – FV Example 6.3

- A) Suppose you plan to deposit $100 into an account in one year and $300 into the account in the third year. How much will be in the account in five years if the interest rate is 8%?

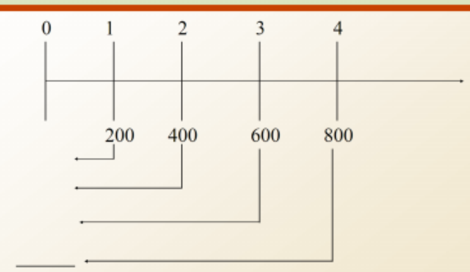
- B) Suppose you plan to deposit $100 into an account in one year and $300 into the account in the fifth year. How much will be in the account in five years if the interest rate is 8%?

- C) Suppose you plan to deposit $100 into an account in one year and $300 in the next four year. How much will be in the account in five years if the interest rate is 8%?

Multiple Cash Flows – Present Value Example 6.4

- You are offered an investment that will pay you $200 in one year, $400 the next year, $ 600 the next year, and $800 at the end of fourth year. You can earn 12 % on very similar investments.

Timeline 6.4

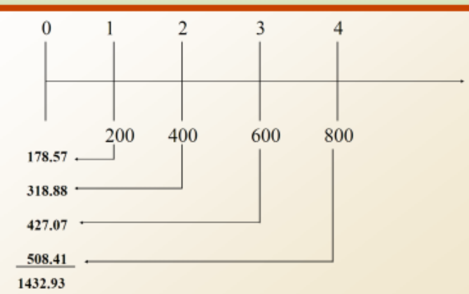


Answer 6.4

- Find the PV of each cash flows and add them
  - Year 1 CF: 200 / $(1.12)^1$ = 178.57
  - Year 2 CF: 400 / $(1.12)^2$ = 318.88
  - Year 3 CF: 600 / $(1.12)^3$ = 427.07
  - Year 4 CF: 800 / $(1.12)^4$ = 508.41

  Total PV = 178.57 + 318.88 + 427.07 + 508.41 =1432.93

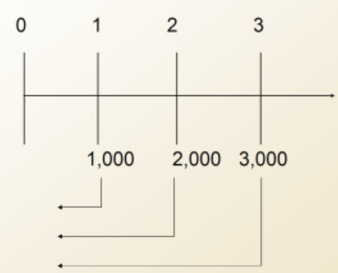
Timeline 6.4



Multiple Cash Flows – PV Another Example 6.5

- You are considering an investment that will pay you $1000 in one year, $2000 in two years and $3000 in three years. If you want to earn 10% on your money, how much would you be willing to pay?

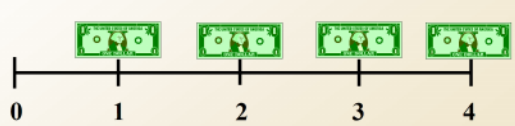
Timeline 6.5



Answer 6.5

Annuities

- **Annuity: a sequence of equal cash flows, occurring at the end of each period.**



Examples of Annuities:

- **If you buy a bond, you will receive equal coupon interest payments over the life of the bond.**
- **If you borrow money to buy a house or a car, you will pay a stream of equal payments.**

Annuities – Basic Formula

- Annuity – equal payments that occur at regular intervals

$$PV = A\left[\frac{1 - \frac{1}{(1+r)^t}}{r}\right]$$

$$FV = A\left[\frac{(1+r)^t - 1}{r}\right]$$

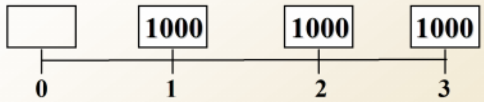
- Notice that the calculator solution is slightly different because of rounding errors in the tables.

Annuities and Perpetuities – Basic Formula

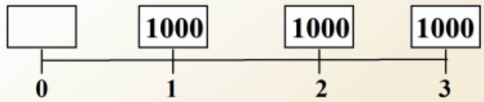
- Perpetuity – infinite series of equal payments
- Special case of an annuity arises when the level of stream of CF (month, year etc.) continues forever.

$$PV = \frac{A}{r}$$

Example 6.6 : Future Value - annuity :If you invest $1,000 at the end of the next 3 years, at 8%, how much would you have after 3 years?



Example 6.7: Present Value - annuity What is the PV of $1,000 at the end of each of the next 3 years, if the opportunity cost is 8%?

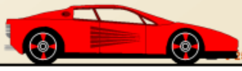


Example 6.8:

What should you be willing to pay in order to receive $10,000 annually forever, if you require 8% per year on the investment?

Annuity – Example 6.9

- You have determined that you can afford to pay $632 per month toward a new Italian sports car. You call up your bank and find out that the going rate is 1% per month for 50 months. How much can you borrow?



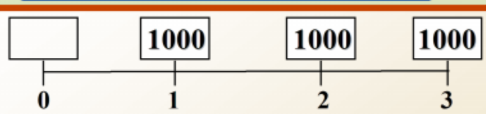
Answer 6.9

Example 6.10: Future Values for Annuities

- Suppose you begin saving for your retirement by depositing $2000 per year in an IRA. If the interest rate is 7.5%, how much will you have in 40 years?

Answer 6.10

Earlier, we examined this "ordinary" annuity:



Using an interest rate of 8%, we find that:

- The **Future Value** (at 3) is $3,246.40.
- The **Present Value** (at 0) is $2,577.10.

What about this annuity?

1000 | 1000 | 1000 | [ ]

0 — 1 — 2 — 3

- Same 3-year time line,
- Same 3 $1000 cash flows, but
- The cash flows occur at the beginning of each year, rather than at the end of each year.
- This is an *"annuity due."*

Future Value - annuity due

If you invest $1,000 at the beginning of each of the next 3 years at 8%, how much would you have at the end of year 3?

1000 | 1000 | 1000 | [ ]

0 — 1 — 2 — 3

Mathematical Solution:

Simply compound the FV of the ordinary annuity one more period:

Mathematical Solution: Simply compound the FV of the ordinary annuity one more period:

$$FV = A\ (FVIFA_{i,\,n})\ (1 + i)$$

$$FV = 1{,}000\ (FVIFA_{.08,\,3})\ (1.08)$$

$$FV = A \left[\frac{(1 + r)^n - 1}{r}\right](1 + r)$$

$$FV = 1{,}000 \left[\frac{(1.08)^3 - 1}{.08}\right](1.08) = 3{,}506.11$$

Present Value - annuity due

What is the PV of $1,000 at the beginning of each of the next 3 years, if your opportunity cost is 8%?

1000 | 1000 | 1000 | [ ]

0 — 1 — 2 — 3

Mathematical Solution: Simply compound the FV of the ordinary annuity one more period:

Mathematical Solution: Simply compound the ordinary annuity one more period:

$$PV = A\ (PVIFA_{r,\,n})\ (1 + r)$$

$$PV = 1{,}000\ (PVIFA_{0.08,\,3})\ (1.08)$$

$$PV = A \left[\frac{1 - \frac{1}{(1 + r)^t}}{r}\right](1 + r)$$

$$PV = 1000 \left[\frac{1 - \frac{1}{(1.08)^3}}{0.08}\right](1.08) = 2{,}783.26$$

Uneven Cash Flows

-10,000 | 2,000 | 4,000 | 6,000 | 7,000

0 — 1 — 2 — 3 — 4

- Is this an annuity?
- How do we find the PV of a cash flow stream when all of the cash flows are different? (Use a 10% discount rate).

-10,000 2,000 4,000 6,000 7,000

0 1 2 3 4

period	CF	PV (CF)
0	-10,000	-10,000.00
1	2,000	1,818.18
2	4,000	3,305.79
3	6,000	4,507.89
4	7,000	4,781.09
PV of Cash Flow Stream:		$ 4,412.95

Example 6.11

- **Cash flows from an investment are expected to be $40,000 per year at the end of years 4, 5, 6, 7, and 8. If you require a 20% rate of return, what is the PV of these cash flows?**

Answer 6.11 Timeline

Answer 6.11

Things to Remember

- You ALWAYS need to make sure that the interest rate and the time period match.
  - If you are looking at annual periods, you need an annual rate.
  - If you are looking at monthly periods, you need a monthly rate.

Compounding Intervals

- Suppose that rate is quoted as 10% Compounded Annually, t=5 years; Then FV=?

  $FV=PV\ (1+0.10)^5$
- Compounded Semi-Annually

  r=0.10/2 = 0.05, t=5*2=10

  $FV=PV\ (1+0.05)^{10}$

  Then this means that investment actually pays 5% every 6 months.

Compounding Intervals

- Compound Quarterly

  r=0.10/4=0.025, t=5*4=20

  $FV=PV\ (1+0.025)^{20}$
- Compound Monthly

  r=0.10/12=0.0083, t=5*12=60

  $FV=PV\ (1+0.0083)^{60}$
- Compound Daily

  r=0.10/365=0.00027, t=5*365=1825

Example 6.12 :

- **After graduation, you plan to invest $400 per month in the stock market. If you earn 12% per year on your stocks, how much will you have accumulated when you retire in 30 years?**

Answer 6.12

House Payment Example 6.13

If you borrow $100,000 at 7% fixed interest for 30 years in order to buy a house, what will be your monthly house payment?

Answer 6.13

Finding the Payment – Example 6.14

- Suppose you want to borrow $20,000 for a new car. You can borrow at 8% per year compounded monthly. If you take a 4 year loan, what is your monthly payment?

Answer 6.14

APRs and EARs

- Interest rates are quoted in different way, is the way that mislead borrowers and investors.
  - Annual Percentage Rate (APR)
  - Effective Annual Rate (EAR)

APR

- APR is the **nominal interest rate** that is not adjusted for the full effect of compounding. (Also known as nominal annual rate)
- Nominal interest rate ignores the time value of money.
- **APR =**
  Period Rate * the number of periods per year

Annual Percentage Rate

- This is the annual rate that is quoted by law
- If the i.r is quoted in terms of an APR, then this indicates the amount of simple interest earned in 1 year, i.e. the amount of **interest earned without the effect of compounding.**
- As the APR does not include the effect of compounding, the APR quote is typically less than the actual amount of interest that you will earn.
- To compute the acual amount that you will earn in 1 year, the APR must first be converted to an Effective Annual Rate (EAR).

Effective Annual Rate (EAR)

- This is the actual rate paid or received after accounting for compounding that occurs during the year
- When time value of money is taken into consideration, the interest rate is called effective interest rate.
- If you want to compare different investments (or interest rates) with different compounding periods you need to compute the EAR and use that for comparison.

EAR - Formula

$$\mathrm{EAR} = \left[1 + \frac{\mathrm{APR}}{\mathrm{m}}\right]^{\mathrm{m}} - 1$$

Remember that the APR is the quoted rate

m is the number of compounding periods per year

Computing APRs from EARs

- If you have an effective rate, how can you compute the APR? Rearrange the EAR equation and you get:

$$APR = m\left[(1+EAR)^{1/m} - 1\right]$$

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EAR – Example 6.15 a

a) You are looking at two savings accounts. One pays 5.25%, with daily compounding. The other pays 5.3% with semiannual compounding. Which account should you use?

6-55

Answer 6.15 a

Example 6.15 b

- b) Which of this is the best if you are thinking of openning a savings account ? Which of these is best if they represent loan rates?
- Bank A: 15 % compounded daily
  Bank B: 15.5 % compounded quarterly
  Bank C: 16 % compounded annually

6-57

Answer 6.15 b

Example 6.16

- Suppose that a credit card agreement quotes an interest rates of 18% APR. Monthly payments are required. What is the actual interest rate you pay on such a credit card?

6-59

Answer 6.16

6-60

Example 6.17

- You have just received your first credit card and the problem is the rate. It look pretty high to you. The annual percentage rate (APR) is listed at 21.7 percent, and when you look closer, you notice that the interest rate is compounded daily. What is the annual percentage yield, or effective annual rate, on your credit card?

6-61

Answer 6.17

6-62

APR – Example 6.18

- Suppose you want to earn an effective rate of 12% and you are looking at an account that compounds on a monthly basis. What APR must they pay?

6-63

Answer 6.18

6-64

Computing Payments with APRs – Example 6.19

- Suppose you want to buy a new computer system and the store is willing to sell it to allow you to make monthly payments. The entire computer system costs $3500. The loan period is for 2 years and the interest rate is 16.9% compounded monthly. What is your monthly payment?

6-65

Answer 6.19

Future Values with Monthly Compounding – Example 6.20

- Suppose you deposit $50 a month into an account that has an APR of 9% compounded monthly. How much will you have in the account in 35 years?

Answer 6.20

Present Value with Daily Compounding – Example 6.21

- You need $15,000 in 3 years for a new car. If you can deposit money into an account that pays an APR of 5.5%, compounded daily. How much would you need to deposit?

Answer 6.21

Example 6.22

Upon retirement, your goal is to spend 5 years traveling around the world. To travel in style will require $250,000 per year at the beginning of each year.

If you plan to retire in 30 years, what are the equal monthly payments necessary to achieve this goal?

The funds in your retirement account will compound at 10% annually.

Answer 6.22

Answer 6.22

Example 6.23

- Suppose you are looking at the following possible cash flows: Year 1 CF = $100; Years 2 and 3 CFs = $200; Years 4 and 5 CFs = $300. The required discount rate is 7%
- a) What is the value of the cash flows at year 5?
- b) What is the value of the cash flows today?

Answer 6.23 a

Answer 6.23 b

Example 6.24

- You want to receive 5000 per month in retirement. If you can earn 7.5% per month and you expect to need the income for 10 years, how much do you need to have in your account?

Answer 6.24

6-78

Example 6.25

- You want to receive $5000 per month for the next 5 years. How much would you need to deposit today if you can earn 0.75% per month?

6-79

Answer 6.25

6-80

Example 6.26

- Find the present value of the CF provided below given a 6% discount rate.

Year	CF	Year	CF
1	$ 500	6	500
2	200	7	500
3	-400	8	500
4	500	9	500
5	500	10	500

6-81

Answer 6.26 Timeline

Answer 6.26

6-83

Example 6.27

- What is the PV of an investment involving \$200 received at the end of years 1 through 5, a \$300 cash outflow at the end of year 6, and \$500 received at the end of years 7 through 10, given a 5% discount rate?

Answer 6.27 Timeline

Answer 6.27

Table

I. Symbols:

PV = Present value, what future cash flows are worth today

FV_t = Future value, what cash flows are worth in the future

r = Interest rate, rate of return, or discount rate per period—typically, but not always, one year

t = Number of periods—typically, but not always, the number of years

C = Cash amount

II. Future Value of *C* per Period for *t* Periods at *r* Percent per Period:

$FV_t = C \times \{[(1 + r)^t - 1]/r\}$

A series of identical cash flows is called an *annuity*, and the term $[(1 + r)^t - 1]/r$ is called the *annuity future value factor*.

III. Present Value of *C* per Period for *t* Periods at *r* Percent per Period:

$PV = C \times \{1 - [1/(1 + r)^t]\}/r$

The term $\{1 - [1/(1 + r)^t]\}/r$ is called the *annuity present value factor*.

IV. Present Value of a Perpetuity of *C* per Period:

$PV = C/r$

A *perpetuity* has the same cash flow every year forever.

Suggested Problems

- 1-19, 21, 24-28, 30, 32-38, 40, 42-44, 46, 68.